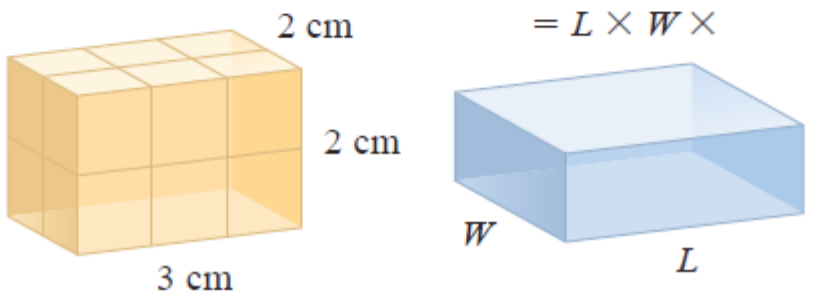
Lecture 17

QR(I) Dr Faraha Ashraf

A › Volume of Rectangular Solids

The volume of a rectangular solid is the number of unit cubes it takes to fill it. In the metric system a solid cube 1 centimeter on each side is defined to be the unit of volume, 1 cm × 1 cm × 1 cm = 1 cm^3 (read "one cubic centimeter"). Other units of volume are the cubic meter and, in the customary system, the cubic foot and cubic yard. (Note that volume is measured in cubic units.)

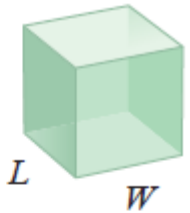
As in the case of areas, the volume of a rectangular solid object equals the number of units of volume it contains. Suppose you have a block 3 centimeters long, 2 centimeters wide, and 2 centimeters high. You can see that the top layer has 3 rows each containing 2 unit volumes, that is, 6 cubic centimeters (cm^3) in all. Since the bottom layer is identical, the volume of this cube is 12 cubic centimeters. This volume can also be found by multiplying 2 cm × 3 cm × 2 cm = 12 cm^3. In general, the volume V of a rectangular solid is found by multiplying the length L times the width W times the height H. This is also the area of the base (L · W) times the height (H) of the solid.



VOLUME OF A RECTANGULAR SOLID

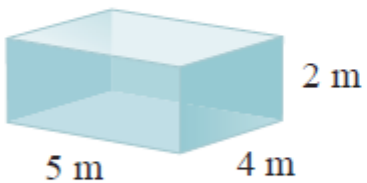
The volume V of a rectangular solid is the product of its length L, its width W, and its height H. In symbols,

$$V = \underbrace{L \cdot W}_{\text{Base area}} \cdot H$$



EXAMPLE 1 Volume of a rectangular solid

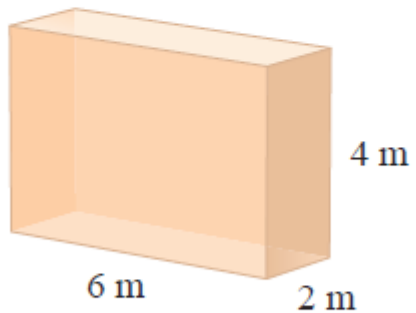
Find the volume of this solid.



SOLUTION The volume is $5\text{ m} \cdot 4\text{ m} \cdot 2\text{ m} = 40\text{ m}^3$.

PROBLEM 1

Find the volume of this solid.



Answers to PROBLEMS

1. 48 m^3

and 12 inches thick, we first have to write all the measurements in yards. Thus,

$$9 \text{ ft} = 3 \text{ yd}$$
$$6 \text{ ft} = 2 \text{ yd}$$
$$12 \text{ in.} = \frac{1}{3} \text{ yd}$$

The volume of the box is

$$V = L \cdot W \cdot H = 3 \text{ yd} \cdot 2 \text{ yd} \cdot \frac{1}{3} \text{ yd} = 2 \text{ yd}^3$$

EXAMPLE 2 Volume of a rectangular space

How many cubic yards of concrete are needed to fill a hole 10 feet long, 6 feet wide, and 18 inches thick?

SOLUTION Recall that 3 ft = 1 yd and 1 ft = 12 inches.

$10 \text{ ft} = 3\frac{1}{3} \text{ yd}$, $6 \text{ ft} = 2 \text{ yd}$, and $18 \text{ in.} = \frac{1}{2} \text{ yd}$

Thus, the volume is

$$V = L \cdot W \cdot H = 3\frac{1}{3} \text{ yd} \cdot 2 \text{ yd} \cdot \frac{1}{2} \text{ yd}$$
$$= 3\frac{1}{3} \text{ yd}^3$$

PROBLEM 2

How many cubic yards of concrete are needed to fill a hole 18 feet long, 4 feet wide, and 9 inches thick?

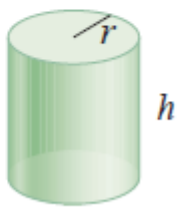
B › Volume of Cylinders

Most soda cans are made of aluminum and are in the shape of a cylinder about 5 inches in height and 2 inches in diameter. How do we find the volume of such a can? The volume of a circular cylinder like the can is found in the same way as the volume of a rectangular solid: it is the product of the area of the base times the height. For a circular cylinder of radius r and height h, the area of the base will be the area of a circle, that is, πr^2. In order to find the volume we multiply the base area (πr^2) by the height h.

VOLUME OF A CIRCULAR CYLINDER

The volume V of a circular cylinder of radius r and height h is the product of π, the square of the radius r, and its height h. In symbols,

$$V = \underbrace{\pi \cdot r}_{\text{Base area}} \cdot h$$

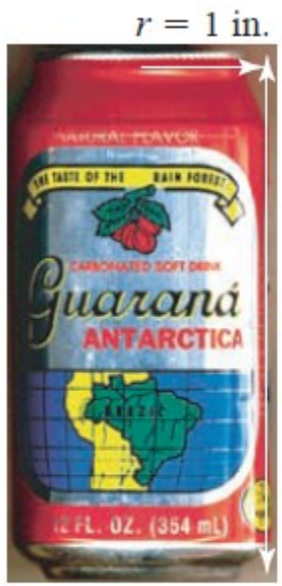


EXAMPLE 3 Volume of a cylinder

Find the volume of the can. Use 3.14 for π.

SOLUTION The can is in the shape of a circular cylinder, and its volume is

$$\begin{aligned} V &= \pi \cdot r^2 \cdot h \\ &= 3.14 \cdot (1 \text{ in.})^2 \cdot (5 \text{ in.}) \\ &= 15.7 \text{ in.}^3 \end{aligned}$$



PROBLEM 3

Find the volume of a can 6 inches high with a radius of 1 inch. Use 3.14 for π.

Answers to PROBLEMS

2. 2 yd^3 **3.** 18.84 $in.^3$

C › Volume of Spheres

The two bowling balls are shaped like **spheres.** A sphere is a three-dimensional solid that is defined as the set of all points in space that are a given distance (the radius r) from its center. The volume of a sphere of radius r is given by the following formula.

VOLUME OF A SPHERE

The volume V of a sphere of radius r is the product of $\frac{4}{3}\pi$ and the radius cubed. In symbols,

$$V = \frac{4}{3}\pi \cdot r^3$$

EXAMPLE 4 Volume of bowling ball

The radius of a bowling ball is 10.9 centimeters. Find the volume of the ball. Use 3.14 for π and round the answer to the nearest hundredth of a cubic centimeter.

SOLUTION The volume of a sphere is $V = \frac{4}{3} \cdot \pi \cdot r^3$

$$= \frac{4}{3} \cdot 3.14 \cdot (10.9 \text{ cm})^3$$

$$= \frac{4 \cdot 3.14 \cdot 1295.029}{3} \text{ cm}^3$$

$$\approx 5421.85 \text{ cm}^3$$

PROBLEM 4

Find the volume of a bowling ball that is about 4.3 inches in diameter. Use 3.14 for π and round the answer to the nearest hundredth of a cubic inch.

EXAMPLE 7 Volume of soil removed

How much soil was removed from the portion of the hole shown? Give the answer in cubic yards.

Here, personnel use heavy equipment to remove contaminated soils from a site. EPA conducts removal actions at long-term Superfund sites to address immediate threats to human health or the environment.

SOLUTION Since the answer has to be in cubic yards, we assume that the hole is a rectangular box 20 yards long, 2 yards wide, and 1 yard deep.

(Note: 6 ft = 2 yd and 3 ft = 1 yd.)

The volume of a rectangular solid is $V = L \cdot W \cdot H$

$$= (20 \text{ yd}) \cdot (2 \text{ yd}) \cdot (1 \text{ yd})$$

$$= 40 \text{ yd}^3$$

This is the capacity of two large dump trucks!

EXAMPLE 8 Volume of a capsule

The capsules are in the shape of a cylinder 12 millimeters long with two half spheres with a diameter of 2 millimeters at each end. What is the volume of the capsule? Use 3.14 for π and round the answer to the nearest hundredth of a cubic millimeter.

SOLUTION We have to find the volume V of the cylinder plus the volume of the two half spheres (which make up one whole sphere).

The volume of the cylinder is $V = \pi \cdot r^2 \cdot h$

Here,

$r = 1$ mm and $h = 12$ mm, so $V = 3.14 \cdot (1 \text{ mm})^2 \cdot (12 \text{ mm})$

$= 37.68 \text{ mm}^3$

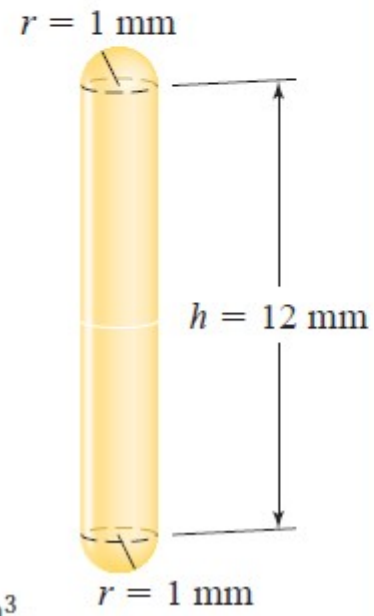
The volume of the sphere is $V = \frac{4}{3} \cdot \pi \cdot r^3$

$= \frac{4}{3} \cdot 3.14 \cdot (1 \text{ mm})^3$

$= \frac{4 \cdot 3.14 \cdot 1}{3} \text{ mm}^3$

$\approx 4.19 \text{ mm}^3$

The volume of the whole capsule is $37.68 + 4.19 = 41.87 \text{ mm}^3$.



PROBLEM 8

Suppose the capsules are in the shape of a cylinder 10 millimeters long with two half spheres of diameter 2 millimeters at each end. What is the volume of each capsule? Use 3.14 for π and round the answer to the nearest hundredth of a cubic millimeter.

Answers to PROBLEMS

8. 35.59 mm^3

Exercises 8.4

MathZone

- Practice Problems
- NetTutor
- Self-Tests
- e-Professors
- Videos

A Volume of Rectangular Solids

In Problems 1–4, find the volume of the rectangular solid with the indicated dimensions.

	Length	Width	Height
1.	8 cm	5 cm	6 cm
2.	15 cm	20 cm	9 cm

	Length	Width	Height
3.	$9\frac{1}{2}$ in.	3 in.	$4\frac{1}{2}$ in.
4.	$5\frac{1}{4}$ in.	$1\frac{3}{8}$ in.	$\frac{5}{8}$ in.

5. The Pet Kennel measures 26 inches long, 19 inches high, and 16 inches wide. What is the volume of the Pet Kennel?

6. The Pet Taxi is 16 inches long, 10 inches high, and 11 inches wide. What is the volume of the Pet Taxi?

7. The inside dimensions of the toaster are 11 inches wide, 8 inches deep, and 5 inches high. What is the volume of the inside of the toaster?

8. The inside dimensions of the microwave are 16 inches by 11 inches by 13 inches. What is the volume of the inside of the microwave?

‹ B › Volume of Cylinders

In Problems 15–20, find the volume of a cylinder with the given radius and height. (Use 3.14 for π and round the answer to the nearest tenth.)

	Radius	Height
15.	10 in.	8 in.
16.	4 in.	18 in.

	Radius	Height
17.	10 cm	20 cm
18.	3.5 cm	2.5 cm

	Radius	Height
19.	1.5 m	4.5 m
20.	0.8 m	3.2 m

21. Find the volume of the large 4-foot high, 1-foot diameter cylinder. Use 3.14 for π and round the answer to the nearest hundredth.

22. Find the volume of the small 2-foot high, 1-foot diameter cylinder. Use 3.14 for π and round the answer to the nearest hundredth.

23. Each sugar container is 5 inches high and 3 inches in diameter. Find the volume of one container. Use 3.14 for π and round the answer to the nearest hundredth.

24. What is the volume of the sugar in the three containers? Note that one of the containers is half-full. Use 3.14 for π and round your answer to the nearest whole number.

‹ C › Volume of Spheres

In Problems 29–32, use 3.14 for π and round the answer to the nearest tenth.

29. A spherical water tank has a 24-foot radius. How many gallons of water will it hold if 1 cubic foot holds about 7.5 gallons?

30. A spherical water tank has a 7.2-meter radius. How many liters of water will it hold if 1 cubic meter holds 1000 liters?

31. The fuel tanks on some ships are spheres, of which only the top halves are above deck. If one of these tanks is 120 feet in diameter, how many gallons of fuel does it hold ($1 \text{ ft}^3 = 7.5$ gal)?

32. The Christmas ornament has a 2-inch diameter. What is its volume?